



SHENTON COLLEGE

Mathematics Applications Year 12 ATMAA

Test 3 2020

Calculator Assumed. 1 page of notes – both sides.

NAME: Solutions (Updated)

Circle Teacher: Cheshire Le McRae Ryan

Time: 50 minutes

Marks: 56

Units (-1)  Rounding (-1)

Show all working in the spaces provided.

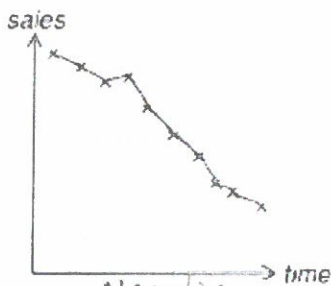
Full marks may not be awarded without sufficient working.

Question 1

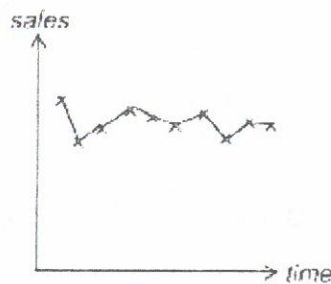
[3 marks: 1, 1, 1]

Examine the following time series and match each plot with one of the following data patterns.

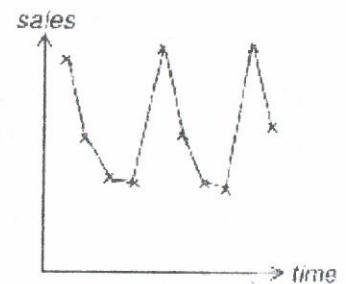
- (i) increasing secular trend
- (ii) decreasing secular trend
- (iii) cyclic
- (iv) seasonal trend
- (v) random trend



(a) Negative Secular Trend ✓



(b) (v) Random ✓



(c) cyclic OR Seasonal Trend (iv) ✓  
[2 marks: 1, 1]

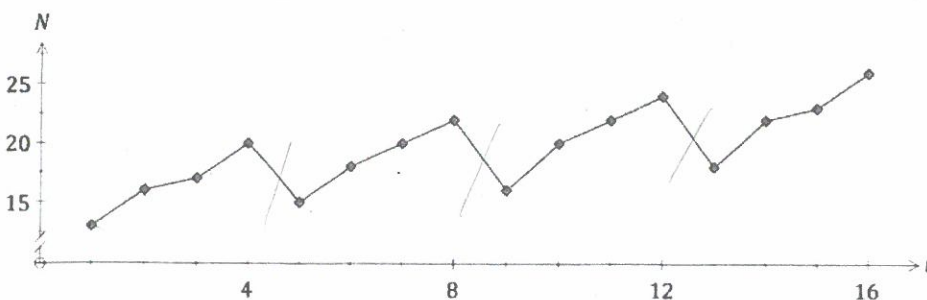
Question 2

Examine the following time series shown below, then determine the most appropriate moving average which should be used to smooth the data.

(a) Moving Average: 3 ✓

Time Period	1	2	3	4	5	6	7	8	9	10
Visitors (000's)	24	28	19	17	21	15	10	18	12	10

(b) Moving Average: 4 ✓



(5)

**Question 3**

[16 marks: 4, 1, 4, 7]

Each year, Australia exports tens of thousands of dairy cattle (cows) and breeding animals to countries all over the world to increase and improve herds through breeding programs. As the impact of COVID-19 continues to evolve, the Australian Livestock Exporters' Council are focused on managing and minimising the impact that government actions and responses will have on the continuity and operation of this export industry. Data of the three previous years is given below.

Time Period (t)	Year	Quarter	Value of exports (\$million)	4pt CMA	Yearly mean	Percentage of yearly mean
1	2016	March	A		191.75	104.3
2		June	180			93.9
3		September	192	192.375		100.1
4		December	195	194.25		101.7
5	2017	March	205	195.875	196.5	D
6		June	190	196.375		96.7
7		September	195	198.375		99.2
8		December	196	B		99.7
9	2018	March	220	200.25	C	109.3
10		June	190	200.75		94.4
11		September	195			96.9
12		December	200			99.4

- (a) Calculate the values of A, B, C and D from the above table. *should do all of these on CA, as 1 mark for each, ie, NO marks for working-out*

A:  $\frac{0.5(A) + 180 + 192 + 195 + 0.5(205)}{4} = 192.375$  B:

A = 200 ✓

200.25 ✓

OR  $\frac{A}{191.75} = 1.043 = 199.975 = 200$

C:  $\frac{220 + 190 + 195 + 200}{4} = C$

D:  $\frac{205}{196.5} = D$

C = 201.25 ✓

D = 104.3 ✓

Accept: 201.28, 201.24, 201.27, 201.21

Accept 104.4, Accept 104.33 & 104

- (b) Calculate the seasonal index for September and write this value in the table below.

Quarter	March	June	September	December
Seasonal Index (%)	106.0	95.0	98.7 ✓	100.3

NO working-out Required → 1 mark.

$106 + 95 + \text{Sept} + 100.3 = 400$

Sept = 98.7

Do not accept 98.73  
Determine SI for Sept. using info for that question i.e. SI table

5 2

- (c) (i) Calculate the deseasonalised value for June 2017.

$$= \frac{\text{Actual}}{SI} = \frac{190}{0.95} = \$200 \text{ Mill answer}$$

↑  
Division (Divide by SI)

- (ii) Comment on the seasonal index found for June.

Export values for June '17 tend to be 5%  
[below the quarter average]

- (d) The equation of the least-squares regression line for the deseasonalised value against time is

$$D = 1.04t + 189.73$$

- (i) Determine whether the dairy cattle and breeding animals are increasing or decreasing with time. Justify your answer with reference to the regression line. (2 marks)

Increasing Trend ✓ • must have increasing Trend, or similar positive gradient of 1.04 ✓

• must state gradient of 1.04 (increase or +ve)

- (ii) Forecast the export value for June 2021. (3 marks)

$$D_{t=22}$$

$$\hat{D} = \$212.61 \text{ Million}$$

• t = 22 ✓

• correct value ✓

• correctly multiply by SI to give answer ✓

Factor back in the seasonality:  $212.61 \times 0.95 = \$201.98 \text{ Million}$   
(201 979 500)

- (iii) Comment on the reliability of your forecast. (2 marks)

NOT reliable, ✓ answer with a valid reason  
Extrapolation - outside one cycle (year) ✓ state 'outside one cycle'

#### Question 4

[3 marks]

At the beginning of 2020, an electrical plant was purchased for \$2.76 million. Financial consultants estimate that the plant will depreciate by 7.5% each year. Using the reducing balance method, determine the estimated value of this plant at the beginning of 2025.

Give your answer to the nearest thousand dollars.

$$2020 \rightarrow 2025 = 5 \text{ yrs}$$

• 5 yrs ✓

• Value, to 2 dp ✓

• Nearest \$1000 ✓

SEQUENCE OR

$$T_{n+1} = 0.925 T_n, T_0 = 2760000$$

$$T_5 = 1869036.3$$

$$\approx \$1869000$$

$$\text{Value} = P(1 - r)^T$$

$$= 2760000(1 - 0.075)^5$$

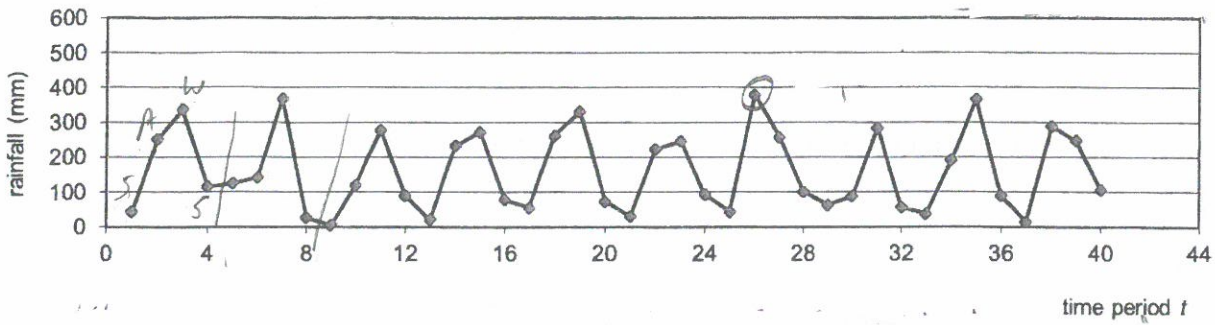
$$= 1869036.36 \checkmark$$

(14)

**Question 5**

[5 marks: 2, 2, 1]

The average rainfall, in millilitres, is recorded for each season over a ten-year period. The given time series plot below shows the rainfall amount recorded during time period  $t$ .



(a) Describe the seasonality and trend of the time series.

Period of 4  
 4 seasons in 1 cycle (year) ✓ Describes 4 pt. (4 quarters)  
 Trend shows a steady pattern (accept steady)  
 Peaks in Winter and troughs in Summer ✓ Describes Trend

\* (b) Explain the purpose of using the moving averages technique for time series data.

To smooth the time series data ✓ smooth data  
 Removes some of the random fluctuations and seasonal effects to identify any long-term trends ✓ observe Trends

(c) Explain the reasoning of centring a 4-point moving average.

Align with time ✓

**Question 6**

[3 marks]

Doctor Ryan Grug invested \$4000 in a savings account, earning interest that is compounded monthly.

At the end of 5 years, the investment account had grown to \$5007.18.

Determine the annual interest rate of Doctor Grug's investment, to two decimal places.

$$5007.18 = 4000 \left(1 + \frac{R}{12}\right)^{60}$$

Contextual. ✓ process 50, -1 for this question

Finance CP  
 DR N: 5 yr  
 I: 4.499  
 PV: -4000  
 PMT: 0  
 FV: 5007.18  
 P/Y: 12  
 C/Y: 12

$R = 0.04499$   
 $\uparrow = 4.50\%$  ✓  
 4.50% pa

4.5% is NOT to 2 dp.

(8)

Question 7

[13 marks: 2, 2, 3, 6]

The spreadsheet below shows the progress of a \$40 000 loan for the first nine months.

Month (n)	Amount owed at the start of the month	Interest charged for the month	Repayment	Amount owed at the end of the month
1	40000.00	300.00	600	39700.00
2	39700.00	297.75	600	39397.75
3	39397.75	$\times 6.09$ $\frac{12}$ B 295.48	600	C 39093.23
4	A	293.20	600	38786.43
5	38786.43	290.90	600	38477.33
6	38477.33	288.58	600	38165.91
7	38165.91	286.24	600	37852.16
8	37852.16	283.89	600	37536.05
9	37536.05	281.52	600	37217.57

(a) Show mathematically that the yearly interest rate is 9%.

$$\frac{300}{40000} \times 100 = 0.75\% \text{ AM}, \therefore \text{Yearly: } 0.75\% \times 12 = 9\%$$

(b) Write a recursive relation to determine the value of this loan at the end of each month.

No marks for working backwards, 0 marks for working backwards

$$T_{n+1} = 1.0075T_n - 600, T_0 = 40000$$

OR  $T_{n+1} = \left(1 + \frac{0.09}{12}\right)T_n - 600$

✓ correct rule  
✓  $T_0 = 40000$

(c) Determine the values of A, B and C in the table above.

A: \$39093.23 ✓      B: \$295.48 ✓      C: \$39093.23 ✓

Do not accept \$39093.33  
( $293.20 \div 0.0075$ )

(d) Determine

(i) the number of repayments to fully repay the loan

$\therefore 93$  Repayments

(ii) the amount of the final repayment

$456.44 \times 1.0075 = \$459.86$

(iii) the total interest charged over the life of the loan, to the nearest dollar

Mortgage:  $\frac{100}{100} \times 1.0075$

$\sum INT = 15659.86$   
or \$15660

OR

Number of repayments + final repayment - Initial value

$$= 92 \times 600 + 459.86 - 40000$$

$$= \$15659.86 \rightarrow \$15660$$

A mortgisation (2 marks)

pm1  
pm2:93  
BAL - 140.14 (2 marks)

OR Finance  
Repayment + BAL  
600 - 140.14  
= \$459.86 (2 marks)

✓ process  
✓ answer

contextual, so (E) for d(iii)

(2 marks)

(2 marks)

(2 marks)

(2 marks)

(2 marks)

(2 marks)

**Question 8**

**[4 marks]**

AnhStrong is considering moving to Denmark, Western Australia from the beginning of September of this year. As there is a much smaller population in Denmark than Perth, AnhStrong believes she will not be as susceptible in catching the COVID virus. AnhStrong investigated all viable options to ensure she would have enough money to move from Perth to Denmark, including funds for her accommodation. Due to her research, AnhStrong is uncertain if she will have enough saved and is also unsure of how much she may need to borrow. She has narrowed her options to two as shown below.

Option A:

McrayFish Quest Loans with an interest rate of 5.20% per annum, compounded monthly.

Option B:

ChesHouse Tennis Bank with an interest rate of 5.30% per annum, compounded quarterly.

Determine which option AnhStrong should choose to maximise her savings and explain your reasoning.

Finance - Interest Conversion

Option A: N 12  
EFF  $\rightarrow$   
APR 5.2

Mcrayfish:  $\left(1 + \frac{0.052}{12}\right)^{12} - 1$   
5.33% ✓  
= 0.053257  
(process = 2 marks)

Option B: N 4  
EFF  $\rightarrow$   
APR 5.3

cheshouse:  $\left(1 + \frac{0.053}{4}\right)^4 - 1$   
5.41% ✓  
= 0.054063

choose Option A ✓ (Mcrayfish) due to Lower interest rate of (5.41 - 5.33) = 0.08%.

✓ Reason

Question 9

[7 marks: 2, 1, 4]

Colin places \$5000 into an investment account which pays 3.9% p.a. where interest is compounded fortnightly. Colin then makes deposits of \$150 at the end of each fortnight.

- (a) Write a recurrence relation to give the value of the investment at the beginning of each fortnight.

$$T_{n+1} = \left(1 + \frac{0.039}{26}\right) T_n + 150 \quad T_1 = \sqrt{5000}$$

$$1.0015 T_n + 150 \quad T_1 = 5000$$

✓ correct recurrence relation

- (b) How much money will be in the account after one fortnight?

FINANCE	
N	1
I%	3.9
PV	-5000
PMT	-150
FV	\$5157.50
P/Y	26
C/Y	26

End of fortnight 1, or start of fortnight 2  
Sequence

$$T_2 = \$5157.50 \checkmark$$

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main

$$1.0015 \times 5000 = \$5157.50$$


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✓ correct answer

- (c) How many weeks will it take for Colin's initial investment to double in value?

Finance	
N	31.04 fortnights
I%	3.9
PV	-5000
PMT	-150
FV	10000
P/Y	26
C/Y	26

∴ 32 fortnights  
= 64 weeks

Sequence

$$T_{32} = \text{start of fortnight 32} = \$9993.97$$

$$T_{33} = \text{start of F/n 33 or end of fortnight 32} = \$10158.96$$

✓ process  
✓ answer in fortnight  
✓ answer in weeks

∴ 32 fortnights  
= 64 weeks

END OF QUESTIONS

(7)

